Chapter 5. Solving Linear Inequations in One Variable

Formulae

Two permissible rules:

1. **Addition – Subtraction Rule:**
   If the same number or expression is added to or subtracted from both sides of an inequation, the resulting inequation has the same solution (or solutions) as the original.
2. **Multiplication – Division Rule:**
   (i) If both sides of an inequation are multiplied or divided by the same positive number, the resulting inequation has the same solution (or solutions) as the original.
   (ii) If both sides of an inequation are multiplied or divided by the same negative number, the resulting inequation has the same solution (or solutions) as the original if the symbol of the inequality is reversed.
   Thus, the only difference between solving a linear equation and solving an inequation concerns multiplying or dividing both sides by a negative number. Therefore, always reverse the symbol of an inequation when multiplying or dividing by a negative number.
3. **Properties of absolute values:**
   (i) $|-x| = |x| \ \forall x \in R$
   (ii) $|xy| = |x|\,|y|, \forall x, y \in R$.
   (iii) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|} \forall x, y \in R \ \& \ y \neq 0$
   (iv) $|x| = \sqrt{x^2} \ \forall x \in R$
   (v) If $a > 0$,
   then $|x| \leq a \Leftrightarrow -a \leq x \leq a$
   $|x| \geq a \Leftrightarrow x \geq a$ or $x \leq -a$.

Determine the Following

Question 1. Give that x ∈ I. Solve the inequation and graph the solution on the number line:

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

Solution :

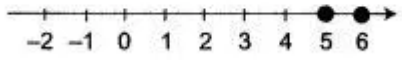
$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

$$\Rightarrow \quad 3 \geq \frac{3(x-4)+2x}{6} \geq 2$$

$$\Rightarrow \quad 18 \geq 5x - 12 \geq 12$$

$$\Rightarrow \quad 30 \geq 5x \geq 24$$

$$\Rightarrow \quad \frac{24}{5} \leq x \leq 6 \qquad (x \in I)$$



∴ The solution set = (5, 6) Ans.

Question 2. Graph the solution set for each inequality:

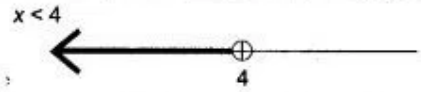
(i) $x \geq -3$ (ii) $x < 4$

(iii) $-3 < x < 5$ (iv) $5 \leq x < 10$

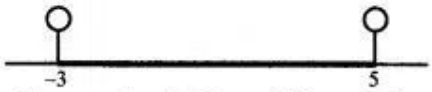
(v) $-3 < x \leq 8$ (vi) $-3 \leq x \leq 3$.

Solution : (i) We shade a number line to the right of – 3. The darkened circle shows – 3 is included.

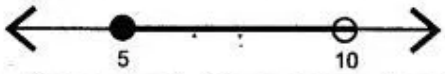
(ii) We shade a number line to the left of 4. The open circle shows that 4 is not included.



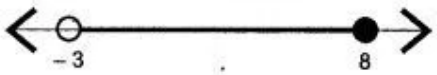
(iii) The graph of $-3 < x < 5$ is all the numbers between 3 and 5.



(iv) The graph of $5 \leq x < 10$ consists of all the numbers between 5 and 10 as well as 5.



(v) The graph of $-3 < x \leq 8$ consists of all the numbers between – 3 and 8 as well as 8.



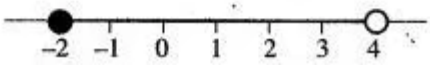
(vi) The graph of $-3 \leq x \leq 3$ consists of all the numbers between – 3 and 3 as well as 3 and – 3.

Question 3. Solve the given inequation and graph the solution on the number line

$2y - 3 < y + 1 \leq 4y + 7, y \in R$

Solution :

$$\begin{aligned} 2y - 3 &< y + 1 \leq 4y + 7 \\ y - 3 &< 1 \leq 3y + 7 \\ y &< 4 \text{ and } 3y \geq -6 \\ y &\geq -2 \\ -2 &\leq y < 4 \end{aligned}$$



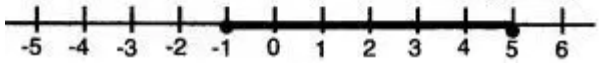
Question 4. Given that x ∈ R, solve the following inequality and graph the solution on the number line:

$-1 \leq 3 + 4x < 23$

Solution :

$$\begin{aligned} -1 &\leq 3 + 4x < 23 \\ -1 - 3 &\leq 3 + 4x - 3 < 23 - 3 \\ -4 &\leq 4x < 20 \\ -\frac{4}{4} &\leq x < \frac{20}{4} \\ -1 &\leq x < 5. \end{aligned}$$

Solution Set $= \{x : -1 \leq x \leq 5, x \in R\}$



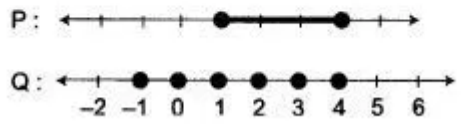
Question 5. Given:

$P = \{x : 5 < 2x - 1 \leq 11, x \in R\}$

$Q = \{x : -1 \leq 3 + 4x < 23, x \in 1\}$

where R = (real number), I = (Integers) Represent P and Q on number lines. Write down the elements of $P \cap Q$.

Solution :



$P \cap Q = \{4\}$.

Question 6. Solve $2 \leq 2x - 3 \leq 5$, $x \in R$ and mark it on a number line.

Solution : $2 \leq 2x - 3 \leq 5, \quad x \in R$

$2 \leq 2x - 3; \quad 2x - 3 \leq 5$

$2 + 3 \leq 2x; \quad 2x \leq 5 + 3$

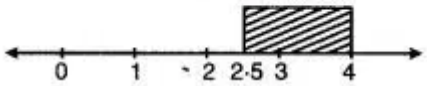
$5 \leq 2x; \quad 2x \leq 8$

or $2x \geq 5; \quad x \leq 4$

or $x \geq \frac{5}{2}$

$\therefore \quad x \geq 2\frac{1}{2}$ and $x \leq 4$

(solution set is $2{\cdot}5 \leq x \leq 4$)



Question 7. For each inequality, determine which of the given numbers are in the solution set:

(i) $2x + 3 > 11$; $-3, 4, 5, 7$

(ii) $16 - 5x \leq -4$; $4, -3, 10$.

Solution : (i) If $x = -3$

Then $2x + 3 = 2 \times (-3) + 3 = -3$

Since, $-3 > 11$ is false.

So -3 is not in the solution of $2x + 3 > 11$

If, $x = 4$, then $2x + 3 = 2 \times 4 + 3 = 11$

since $11 > 11$ is false.

So, 4 is not in the solution of $2x + 3 > 11$

if $x = 5$, then $2x + 3 = 2 \times 5 + 3 = 13$

Since, $13 > 11$ is true :

So, 5 is in the solution of $2x + 3 > 11$

Similarly, $x = 7$ is in the solution of $2x + 3 > 11$.

Ans.

(ii) If $x = 4$, then $16 - 5x = 16 - 5 \times 4 = -4$

Since, $-4 \leq -4$ is true.

So, $x = 4$ is in the solution of $16 - 5x \leq -4$

if $x = -3$, then $16 - 5x = 16 - 5 \times -3 = 31$

Since, $31 \leq -4$ is false.

So, $x = -3$ is not in the solution of $16 - 5x \leq -4$

if $x = 10$, then $16 - 5x = 16 - 5 \times 10 = -34$

Since, $-34 \leq -4$ is true.

So, $x = 10$ is in the solution of $16 - 5x \leq -4$.

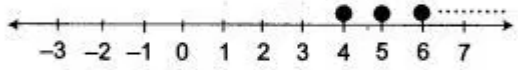
Question 8. Graph the solution sets of the following inequalities:

(i) $2x - 4 > 3, x \in W$

(ii) $3x - 5 \leq -7, \ x \in I.$

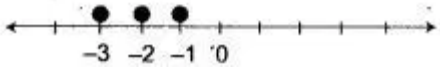
Solution : (i) $2x - 4 > 3$

$$2x > 3 + 4 \Rightarrow 2x > 7$$
$$x > 7/2 \Rightarrow x > 3{\cdot}5$$
$$x = \{4, 5, 6, \ldots\ldots\}$$



(ii) $3x - 5 \leq -7, x \in I$

$$3x < 5 + (-7)$$
$$3x \leq -2$$
$$x \leq -2/3$$
$$x = \{\ldots\ldots, -3, -2, -1\}$$



Question 9. Solve the equation and represent the solution set on the number line.

$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x$, where $x \in I$

Solution :

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \ x \in I,$$

$$-3 + x \leq \frac{8x}{3} + 2, \quad \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x$$

or $\frac{8x}{3} - x \geq -3 - 2, \quad \frac{8x}{3} - 2x \leq \frac{14}{3} - 2$

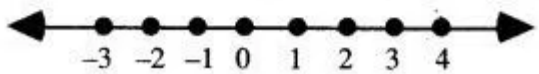
or $\frac{5x}{3} \geq -5, \quad \frac{2x}{3} \leq \frac{8}{3}$

$5x \geq -15, \quad 2x \leq 8$

$x \geq -3, \quad x \leq 4$

Solution set $\{-3, -2, -1, 0, 1, 2, 3, 4\}$ Ans.

Number line



Question 10. Solve the following inequation and represent the solution set on the number line:

$4x-19<\frac{3x}{5}-2\le\frac{-2}{5}+x, x\in R$

Solution : $4x-19<\frac{3x}{5}-2\le\frac{-2}{5}+x, x\in R$

$\therefore \quad 4x-19<\frac{3x}{5}-2,$

$4x-\frac{3x}{5}<-2+19,$

$\frac{17x}{5}<17,$

$x<5,$

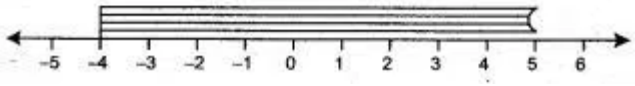
and $\frac{3x}{5}-2\le\frac{-2}{5}+x$

$\frac{3x}{5}-x\le\frac{-2}{5}+2$

$-2x\le 8$

$x\ge -4$

$\Rightarrow \quad -4\le x<5$



Question 11. Solve the following in equalities and graph their solution set

$A = \{x : 11x - 5 \ge 7x + 3, x \in R\}$ and

$B = \{x : 18x - 9 \ge 15 + 12x, x \in R\}$

Solution : $A = \{x : 11x - 5 \ge 7x + 3, x \in R\}$

$= \{x : 11x - 7x \ge 3 + 5, x \in R\}$

$= \{x : 4x \ge 8, x \in R\}$

$= \{x : x \ge 2, x \in R\}$ ...(i)

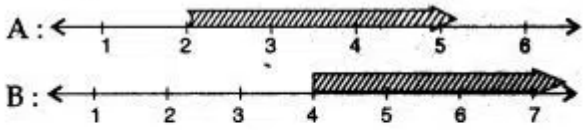
Also $B = \{x : 18x - 9 \ge 15 + 12x, x \in R\}$

$= \{x : 18x - 12x \ge 15 + 9, x \in R\}$

$= \{x : 6x \ge 24, x \in R\}$

$= \{x : x \ge 4, x \in R\}$ ...(ii)

$\therefore$ on number line,



$\therefore A \cap B : \{x : x \ge 4, x \in R\}$

i.e. $A \cap B$:

Question 12. Solve the following inequation and graph the solution set,

(i) $2x-3 \le x+2 \le 3x+5\ x \in R$.

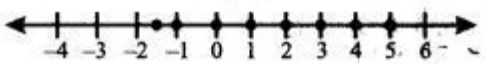
(ii) $2x-5 \le 5x+4 < 11\ n \in R$.

Solution : (i) Here, $2x-3 \le x+2 \le 3x+5$

$\Rightarrow \quad 2x-3 \le x+2$ and $x+2 \le 3x+5$

$\Rightarrow \quad x \le 5$ and $x \ge \frac{-3}{2}$

$\therefore$ Solution set $= \left\{x : \frac{-3}{2} \le x \le 5, \text{and } x \in R\right\}$.



Ans.

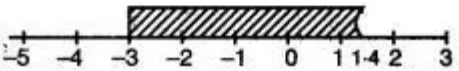
(ii) Here, $2x-5 \le 5x+4 < 11$

$\Rightarrow \quad 2x-5 \le 5x+4$ and $5x+4 < 11$

$\Rightarrow \quad -3x \le 9$ and $5x < 7$

$\Rightarrow \quad x \ge -3$ and $x < \frac{7}{5}$

$\therefore$ Solution set $= \left\{x : -3 \le x < \frac{7}{5} \text{ and } x \in R\right\}$.



Ans.

Question 13. Solve the following inequation and graph the solution on the number line.

$$-2\frac{2}{3} \le x+\frac{1}{3} < 3\frac{1}{3};\ x \in R.$$

Solution : The given inequation has two parts :

$$-2\frac{2}{3} \le x+\frac{1}{3} \text{ and } x+\frac{1}{3} < 3\frac{1}{3}$$

$$-\frac{8}{3} \le x+\frac{1}{3} \text{ and } x+\frac{1}{3} < \frac{10}{3}$$

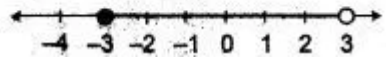
$$-\frac{8}{3}-\frac{1}{3} \le x \text{ and } x < \frac{10}{3}-\frac{1}{3}$$

$$-\frac{9}{3} \le x \text{ and } x < \frac{9}{3}$$

$$-3 \le x \text{ and } x < 3$$

$\therefore \quad -3 \le x < 3$

The required graph line is :



Question 14. Solve the following inequalities and represent the solution on a number line:

(i) $2x + 3 < 5$ (ii) $3x + 4 \le x + 8$

(iii) $2x - 3 > 5x + 4$ (iv) $4 - 2x \ge 6 - 3x$

(v) $3(x - 2) > 1$ (vi) $\frac{2x+5}{4} > \frac{4-3x}{6}$

(vii) $\frac{3x}{2} + \frac{1}{4} > \frac{5x}{8} - \frac{1}{2}$.

Solution : (i) We have, $2x + 3 < 5$

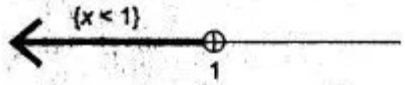
$\Rightarrow \quad 2x < 5 - 3$

$\Rightarrow \quad 2x < 2$

$\Rightarrow \quad x < 1$

The graph of the solution set is

$\{x < 1\}$



Ans.

(ii) We have, $3x + 4 \le x + 8$

$\Rightarrow \quad 3x - x \le 8 - 4$

[Bring like terms on one side]

$\Rightarrow \quad 2x \le 4$

$\Rightarrow \quad x \le 2$

The graph of the solution set is

$x \le 2.$

Ans.

(iii) We have the inequality

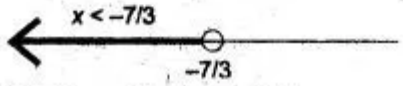
$2x - 3 > 5x + 4$

$\Rightarrow \quad -3 - 4 > 5x - 2x$

$\Rightarrow \quad -7 > 3x \text{ or } x < -7/3$

The graph of the solution set is

$x < -7/3.$



(iv) We have the inequality

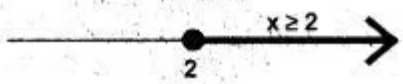
$4 - 2x \ge 6 - 3x$

$\Rightarrow \quad 3x - 2x \ge 6 - 4$

$\Rightarrow \quad x \ge 2$

The graph of the solution set is

$x \ge 2.$

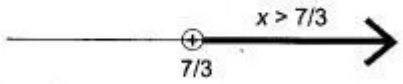


(v) The given inequality is

$$3(x-2) > 1$$
$$\Rightarrow \quad 3x-6 > 1$$
$$\Rightarrow \quad 3x > 7 \Rightarrow x > 7/3$$

The graph of the solution set is given by

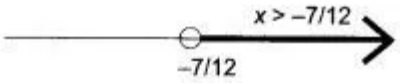
$$x > 7/3.$$



(vi) The given inequality is

$$\frac{2x+5}{4} > \frac{4-3x}{6}$$
$$\Rightarrow \quad 6(2x+5) > 4(4-3x)$$
$$\Rightarrow \quad 12x+30 > 16-12x$$
$$\Rightarrow \quad 12x+12x > 16-30$$
$$\Rightarrow \quad 24x > -14$$
$$\Rightarrow \quad x > \frac{-14}{24}$$
$$x > -7/12.$$

The graph of solution set is $x > -7/12$

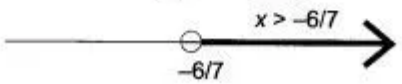


(vii) The given inequality is

$$\frac{3x}{2}+\frac{1}{4} > \frac{5x}{8}-\frac{1}{2}$$
$$\Rightarrow \quad \frac{3x}{2}-\frac{5x}{8} > -\frac{1}{2}-\frac{1}{4}$$
$$\Rightarrow \quad \frac{12x-5x}{8} > \frac{-2-1}{4}$$
$$\Rightarrow \quad \frac{7x}{8} > \frac{-3}{4} \Rightarrow 4(7x) > -3 \times 8$$
$$\Rightarrow \quad 28x > -24$$
$$\Rightarrow \quad x > \frac{-24}{28}$$
$$x > -6/7$$

The graph of the solution set is

$$x > -6/7.$$



Question 15. Solve the following inequalities and represent the solution set on a number line:

(i) $-4 \le 2x-3 \le 5$

(ii) $-3 < -\frac{1}{2}-\frac{2x}{3} \le \frac{5}{6}, x \in R.$

(iii) $0 < \frac{3x-2}{4} \le 2$

(iv) $0 \le \frac{3-2x}{4} \le 1$

(v) $3 > \frac{2(3-4x)}{7} \ge -2.$

Solution : (i) The given inequality

$$-4 \le 2x-3 \le 5$$

is equivalent to

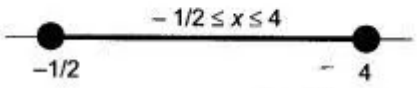
$$3-4 \le 2x \le 5+3$$

$$\Rightarrow \quad -1 \le 2x \le 8$$

$$\Rightarrow \quad -\frac{1}{2} \le x \le 4$$

The graph of this set is

$$-1/2 \le x \le 4.$$



(ii) $$-3 < -\frac{1}{2}-\frac{2x}{3} \le \frac{5}{6}$$

$$\Rightarrow \quad -18 < -3-4x \le 5$$

$$\Rightarrow \quad -15 < -4x \le 8$$

$$\Rightarrow \quad -2 \le x < \frac{15}{4}$$

(iii) The given inequality is

$$0 < \frac{3x-2}{4} \le 2$$

Which is equivalent to

$$0 < 3x-2 \le 8$$

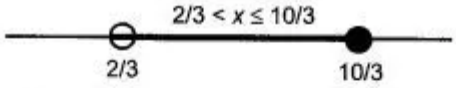
$$\Rightarrow \quad 2 < 3x \le 8+2$$

$$\Rightarrow \quad 2 < 3x \le 10$$

$$\Rightarrow \quad 2/3 < x \le 10/3$$

The graph of this set is

$$2/3 < x \le 10/3.$$



Ans.

(iv) The given inequality is

$$0 \le \frac{3-2x}{4} \le 1$$

same as part (iii) solve yourself.

(v) The given inequality

$$3 > \frac{2(3-4x)}{7} \ge -2$$

which is equivalent to

$$\Rightarrow \quad 3 \times 7 > 2(3-4x) \ge -2 \times 7$$

$$\Rightarrow \quad \frac{21}{2} > 3-4x \ge -7$$

$$\Rightarrow \quad -3+\frac{21}{2} > -4x \ge -7-3$$

$$\Rightarrow \quad \frac{15}{2} > -4x \ge -10$$

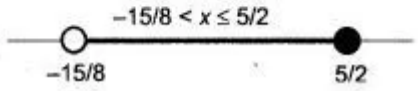
we divide this compound inequality by -4 and reverse the inequality signs to obtain

$$\frac{15}{2 \times (-4)} < x \le \frac{-10}{-4}$$

$$\Rightarrow \quad \frac{15}{-8} < x \leq \frac{5}{2}$$

The graph of this set is

$$\frac{-15}{8} < x \leq \frac{5}{2}.$$



Question 16. Solve the following inequation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in \mathrm{R}$$

Solution : $-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in R$

$$-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3}$$

$$-\frac{x}{3} \leq \frac{x}{2} - \frac{4}{3}$$

$$\frac{4}{3} \leq \frac{x}{2} + \frac{x}{3}$$

$$\frac{4}{3} \leq \frac{5x}{6}$$

$$\frac{6}{5} \times \frac{4}{3} \leq x$$

$$\frac{8}{5} \leq x$$

$$\frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

$$\frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$

$$\frac{x}{2} < \frac{1+8}{6}$$

$$x < \frac{9 \times 2}{6}$$

$$x < 3$$

From (1) and (2)

$$\frac{8}{5} \leq x < 3$$

or $\quad 1{\cdot}6 \leq x < 3$

$\therefore$ Solution set $\{x : 1{\cdot}6 \leq x < 3,\ x \in \mathrm{R}\}$

Question 17. Find the values of x, which satisfy the inequation

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2,\ x \in \mathrm{W}$. Graph the solution set on the number line.

Solution : $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2$

Taking, $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3}$

$$-\frac{17}{6} < \frac{1}{2} - \frac{2x}{3}$$

$$-\frac{17}{6} - \frac{1}{2} < -\frac{2x}{3}$$

$$\frac{-17-3}{6} < -\frac{2x}{3}$$

$$-\frac{20}{6} < -\frac{2x}{3}$$

$$\Rightarrow \frac{10}{3} > \frac{2x}{3}$$

$$5 > x \qquad \ldots(1)$$

Now taking, $\frac{1}{2} - \frac{2x}{3} \le 2$

$$-\frac{2x}{3} \le 2 - \frac{1}{2}$$

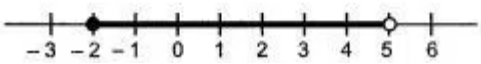
$$-\frac{2x}{3} \le \frac{3}{2}$$

$$-x \le \frac{9}{4} \Rightarrow x \ge -\frac{9}{4} \qquad \ldots(2)$$

From (1) and (2), we get

$$-\frac{9}{4} \le x < 5 \Rightarrow -2\frac{1}{4} \le x < 5$$

Required number line,



Ans.

Question 18. Solve the following inequalities in the given universal set:

(i) $3x - 5 > x + 7; x \in N$

(ii) $4x + 2 \le 2x - 7; x \in I$

(iii) $5x - 3 < 6x - 2; x \in N$

(iv) $2x - 5 \le 5x + 4 < 11$, where $x \in I$.

Solution : (i) We have

$$3x - 5 > x + 7$$

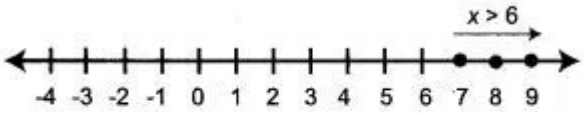
$$\Rightarrow 3x - x > 7 + 5$$

$$\Rightarrow 2x > 12$$

$$\Rightarrow x > 6$$

As $x \in N$, x can take values 7, 8, 9, 10, ...

This set is drawn on the number line as follow.



Ans.

(ii) We have

$$4x + 2 \le 2x - 7; x \in I$$

$$\Rightarrow 4x - 2x \le -7 - 2$$

$$\Rightarrow 2x \le -9$$

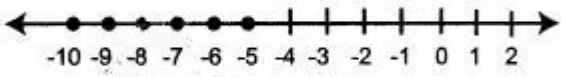
$$\Rightarrow x \le -9/2$$

As $x \in I$, x can take values $-5, -6, -7, \ldots\ldots$,

so $x = \{-, 5, -6, -7, -8, \ldots\ldots\}$

This set can be drawn on number line as

$x \leq -9/2.$



Ans.

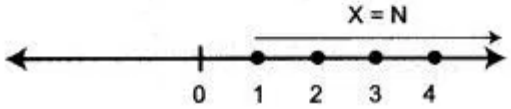
(iii) We have

$$5x - 3 < 6x - 2; x \in N$$
$$\Rightarrow \quad 5x - 6x < -2 + 3$$
$$\Rightarrow \quad -x < 1$$
$$\Rightarrow \quad x > -1$$

As $x \in N$, so x be the set of all natural numbers.

The given set can be represent on number line as

$x = N.$



(iv) $2x - 5 \leq 5x + 4 < 11, x \in I$

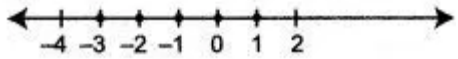
$$2x - 5 \leq 5x + 4$$
$$2x - 5x \leq 4 + 5$$
$$-3x \leq 9$$
$$3x \geq -9$$
$$x \geq -3$$

or $\quad -3 \leq x$

$$5x + 4 < 11$$
$$5x < 11 - 4$$
$$5x < 7$$
$$x < \frac{7}{5}$$
$$x < 1\frac{2}{5}$$

From (1) and (2) $-3 \leq x < 1\frac{2}{5}, x \in I$

$\therefore$ Solution set $= \{-3, -2, -1, 0, 1\}$



Question 19. Find the solution set of the following inequalities and draw the graph of their solutions sets:

(i) $|x + 5| < 8$ (ii) $|x - 1| > 3$

(iii) $|3 - 2x| \geq 2$ (iv) $\left|\frac{x-5}{3}\right| < 6$

(v) $\frac{3}{|x-2|} > 5.$

Solution : (i) We have

$$|x+5| < 8$$

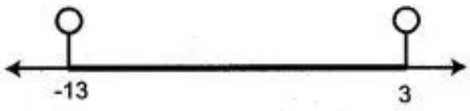
Using prop. $|x| < a \Leftrightarrow -a < x < a$

$$\Rightarrow \quad -8 < x+5 < 8$$

$$\Rightarrow \quad -5-8 < x < 8-5$$

$$\Rightarrow \quad -13 < x < 3$$

The graph of this set is



Ans.

(ii) We have

$$|x-1| > 3$$

Using prop. $|x| \geq a \Leftrightarrow x \geq a$ or $x \leq -a$

Then $|x-1| > 3 \Leftrightarrow x-1 > 3$ or $x-1 < (-3)$

$$\Rightarrow \quad x > 4 \quad \text{or } x < -2$$

so $|x-1| > 3 \{x : x < -2 \text{ or } x > 4\}$

The graph of this set is

(iii) We have

$$|3-2x| \geq 2$$

Using prop. $|x| \geq a \Leftrightarrow x \geq a$ or $x \leq -a$

$$|3-2x| \geq 2$$

or $\quad 3-2x \leq -2$ or $(3-2x) \geq 2$

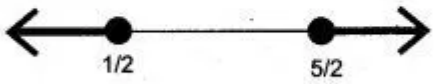
$$\Rightarrow \quad -2x \leq -5 \text{ or } -2x \geq -1$$

$$\Rightarrow \quad 2x \leq 1 \text{ or } 2x \geq 5$$

$$\Rightarrow \quad x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}$$

or $\quad |3-2x| \geq 2 \Rightarrow \left\{x : x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}\right\}$

The graph of this set is Ans.



(iv) We have $\left|\frac{x-5}{3}\right| < 6$

Using prop. $|x| < a \Rightarrow -a < x < a$

$$\left|\frac{x-5}{3}\right| < 6 \Rightarrow -6 < \frac{x-5}{3} < 6$$

$$\Rightarrow -18 < x-5 < 18$$

$$\Rightarrow 5-18 < x < 18+5$$

$$\Rightarrow -13 < x < 23$$

So $\left|\frac{x-5}{3}\right| < 6 \Rightarrow \{x : -13 < x < 23\}$

The graph of this set is

(v) We have $\frac{3}{|x-2|}<5$

$$3 > 5|x-2|$$

$$5|x-2| < 3$$

$$|x-2| < \frac{3}{5}$$

Using property $|x| < a = -a < x < a$

$$\therefore \quad -\frac{3}{5} < x-2 < \frac{3}{5}$$

$$\Rightarrow \quad -\frac{3}{5}+2 < x < \frac{3}{5}+2$$

$$\Rightarrow \quad \frac{7}{5} < x < \frac{13}{5}$$

So $\frac{3}{|x-2|} < 5$

$$\Rightarrow \left\{x : \frac{7}{5} < x < \frac{13}{5}\right\}.$$

The graph of this set is

Question 20. Solve the following inequalities and graph their solution set:

(i) $\frac{2x-5}{x+2}<2$ (ii) $\frac{x+8}{x+1}>1$.

Solution : (i) The inequality $\frac{2x-5}{x+2} < 2$ is equiva-lent to $\frac{2x-5}{x+2}-2<0 \Leftrightarrow \frac{2x-5-2x-4}{x+2}<0$

$$\Rightarrow \quad \frac{-9}{x+2} < 0$$

But $\frac{a}{b} < 0, a<0 \Rightarrow b>0$

Thus, $\frac{-9}{x+2}<0, -9<0 \Rightarrow x+2>0 \Rightarrow x>-2$

The graph of this solution is

$$x > -2.$$

Ans.

(ii) The inequality $\frac{x+8}{x+1}>1$ is equivalent to

$$\frac{x+8}{x+1}-1>0 \Leftrightarrow \frac{x+8-x-1}{x+1}>0$$

$$\Rightarrow \quad \frac{7}{x+1} = 0$$

but $\frac{a}{b} > 0, a>0 \Leftrightarrow b>0$

Thus, $\frac{7}{x+1}>0, 7>0 \Rightarrow x+1>0$ or $x>-1$

The graph of this set is